Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/24**

Paper 2 Paper 24 **May/June 2021**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

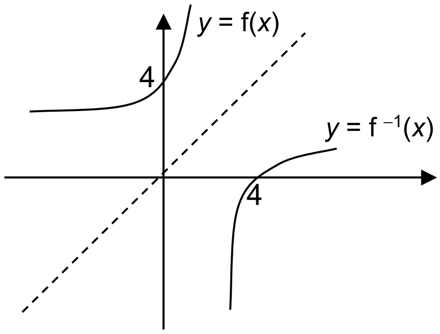
awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	Rearranges to a 3-term quadratic $p^2 + p - 4 = 0$	**M1**	Condone one sign error in coefficients but not powers; powers must be simplified
	Solves *their* 3-term quadratic in p soi	**M1**	
	$p = \dfrac{-1+\sqrt{17}}{2}$ cao; nfww	**A1**	
2	$\dfrac{1}{2}\ln(2x-3)+\dfrac{2}{3}x^{\frac{3}{2}}\ (+c)$ isw or $\dfrac{1}{2}\ln(x-1.5)+\dfrac{2}{3}x^{\frac{3}{2}}\ (+c)$ isw	**B3**	**B2** for $\dfrac{1}{2}\ln(2x-3)$ or $\dfrac{1}{2}\ln(x-1.5)$ or **B1** for $\dfrac{1}{2}\ln 2x-3$ or for $k\ln(2x-3)$ or $k\ln(x-1.5)$ where k is non-zero and $k \neq \dfrac{1}{2}$ and **B1** for $\dfrac{2}{3}x^{\frac{3}{2}}(+\ c)$
3	gradient = $\dfrac{11--4}{2--1}$ oe or 5 soi	**M1**	or $-4=-m+c$ and $11=2m+c$ oe and subtracting/substituting to solve for m or c, condone one error
	$Y-11 = their\ 5(X-2)$ oe or $Y--4 = their\ 5(X--1)$ oe or $11 = 2\times their\ 5+c$ oe or $-4=-1\times their\ 5+c$ oe	**M1**	or using *their m* or *their c* to find *their c* or *their m*, without further error
	$\lg y = 5\lg x + 1$ oe or $\lg y = 5(\lg x - 2)+11$ oe or $\lg y = 5(\lg x --1)-4$ oe	**A1**	
	$y = 10^{5\times\lg x+1}$ oe or $\lg y = \lg x^5 + \lg 10$ oe or $\lg\left(\dfrac{y}{x^5}\right)=1$ oe	**A1**	this first step in manipulating the equation, or the second step, must be seen; only one step may be implied
	$y = 10^{\lg x^5}\times 10^{[1]}$ oe or $\lg y = \lg 10x^5$ oe or $\dfrac{y}{x^5}=10^{[1]}$ oe or $\lg\left(\dfrac{y}{x^5}\right)=\lg 10$	**A1**	this second step in manipulating the equation, or the first step, must be seen; only one step may be implied
	$y = 10x^5$	**A1**	must be seen

Question	Answer	Marks	Partial Marks
3	Alternative $\lg y = \lg a + n \lg x$ soi	**(B1)**	
	Valid method to find n or lg$a$ e.g. Solves for n or lg$a$: $11 = \lg a + 2n$ and $-4 = \lg a - n$	**(M1)**	May find gradient as above (do not need to identify it as n for this mark)
	n = 5 or lg$a$ = 1 so $a = 10^1$ or 10	**(A1)**	
	Correct method using *their* lg$a$ or *their n* to find *their n* or *their* lg$a$	**(M1)**	**FT** May find intercept as above (do not need to identify it as lg$a$ for this mark)
	lg$a$ = 1 so $a = 10^1$ or 10 or n = 5	**(A1)**	
	$y = 10x^5$	**(A1)**	must be seen
4	$\frac{dy}{dx} = 5x^4 - 6x^2 + 2x$	**M2**	Must be seen or very clearly implied by $5(-1)^4 - 6(-1)^2 + 2(-1)$ **M1** for any 2 out of 3 terms correct and no extras or for all 3 terms correct and one extra term
	$\left.\frac{dy}{dx}\right\rvert_{x=-1} = -3$	**A1**	
	gradient of normal = $-\frac{1}{their(-3)}$	**M1**	
	$y = 5$	**B1**	
	$3y = x + 16$ or $y - 5 = \frac{1}{3}(x+1)$ oe, isw	**A1**	**FT** $-\frac{1}{their(-3)}$ and *their* 5, providing *their* $5 \neq 0$
	$P(-16, 0)$	**B1**	

Question	Answer	Marks	Partial Marks
5	Correctly eliminates x or y: $(3y + 20)^2 + y^2 - 2(3y + 20) + 6y = 0$ or $x^2 + \left(\frac{x-20}{3}\right)^2 - 2x + 6\left(\frac{x-20}{3}\right) = 0$	**M1**	
	$10y^2 + 120y + 360[=0]$ or $10x^2 - 40x + 40[=0]$	**A1**	
	Factorises *their* 3-term quadratic or solves *their* 3-term quadratic = 0	**M1**	
	$x = 2$ and $y = -6$	**A1**	
6(a)	$\frac{dy}{dx} = \frac{1}{3}(x^3 - 91)^{-\frac{2}{3}} \times 3x^2$ oe, isw	**B2**	**B1** for $\frac{1}{3}(x^3 - 91)^{-\frac{2}{3}} \times ...$ or for $\frac{1}{3}(.....)^{-\frac{2}{3}} \times 3x^2$ or for $their\,\frac{1}{3}(x^3 - 91)^{\left(their\,\frac{1}{3}\right)-1} \times 3x^2$ or for $k(x^3 - 91)^{-\frac{2}{3}} \times 3x^2$ where k is a constant and $k \neq \frac{1}{3}$
6(b)	$\frac{\delta y}{h} = their\left(\left.\frac{dy}{dx}\right\|_{x=6}\right)$ oe	**M1**	
	$1.44h$ oe, isw	**A1**	dep on **B2** in part (a)
7(a)	$4\left(x - \frac{1}{2}\right)^2 + 6$ isw	**B3**	**B1** for $\left(x - \frac{1}{2}\right)^2$ **B1** for $r = 6$
7(b)	$\frac{1}{their\,6}$	**B1**	**strict FT** an expression of the correct form except, if incorrect form but values of p, q, r stated give BOD using those values; must not be incorrectly attributed to x
	$x = their\,\frac{1}{2}$ oe	**B1**	**strict FT** an expression of the correct form except, if incorrect form but values of p, q, r stated give BOD using those values; must be correctly attributed to x

Question	Answer	Marks	Partial Marks
8(a)(i)	Uses correct Pythagorean identity in the left-hand side of the given identity, e.g. $\dfrac{1-\sin^2 2x}{1+\sin 2x}$	**M1**	
	$\dfrac{(1-\sin 2x)(1+\sin 2x)}{1+\sin 2x}$ oe and completion to given answer	**A1**	
8(a)(ii)	$\sin 2x = \dfrac{2}{3}$	**M1**	
	$x = \dfrac{1}{2}\sin^{-1}\left(\dfrac{2}{3}\right)$ soi	**M1**	dep on first **M1**
	20.9 or 20.905… rounded or truncated to 4 or more figures and 69.1 or 69.094… rounded or truncated to 4 or more figures	**A2**	with no incorrect values in range **A1** for either angle correct, ignoring extra values
8(b)	$\tan\left(y-\dfrac{\pi}{2}\right) = \dfrac{1}{\sqrt{3}}$ soi	**M1**	
	$y = \dfrac{\pi}{6} + \dfrac{\pi}{2}$	**M1**	dep on first **M1**
	$\dfrac{2}{3}\pi$ oe or 2.09 or 2.094[39…] rot to 4 or more significant figs	**A1**	with no incorrect values in range
9(a)	$f > 3$	**B1**	
9(b)	Complete method: Putting $y = f(x)$ and changing subject to x and swapping x and y or swopping x and y and changing subject to y	**M1**	must be a function of x not y
	$f^{-1}(x) = \dfrac{1}{5}\ln(x-3)$	**A1**	
	[Domain] $x > 3$	**B1**	**FT** *their* part (a) providing *their* part (a) is of the form $f > a$ or $f \geqslant a$, where a is a constant

Question	Answer	Marks	Partial Marks
9(c)	*their* $\ln(x-3)=\ln 1$ soi or *their* $(x-3)=\mathrm{e}^0$ soi or $\mathrm{f}(0)=3+\mathrm{e}^{5\times 0}$ soi	**M1**	**FT** *their* $\mathrm{f}^{-1}(x)$ of the form $k\ln(\pm x\pm 3)$, where k is a non-zero constant
	$x=4$	**A1**	
9(d)	Fully correct graph:  	**B4**	**B2** for correct exponential shape for f, in 1st and 2nd quadrants, with correct asymptotic behaviour soi or **B1** for a correct exponential shape for f, in 1st and 2nd quadrants, with asymptotic behaviour but to a clearly incorrect line $y=k$ soi **B1** for f^{-1} reflection of f in the line $y=x$ **B1** for intercepts (0, 4) and (4, 0) and no others; must have attempted correct exponential and logarithmic shapes **Maximum of 3 marks if not fully correct**
10(a)(i)	$v=-\frac{1}{3}\mathrm{e}^t+10\mathrm{e}^{-t}$	**B1**	
	$-\frac{(\mathrm{e}^t)^2}{3}+10=0$ or $30-\mathrm{e}^{2t}=0$ oe	**M1**	**FT** *their* v providing of the form $m\mathrm{e}^t+n\mathrm{e}^{-t}$, where m and n are constants
	$\mathrm{e}^t=\sqrt{30}$ or $2t=\ln 30$ oe	**M1**	dep on previous **M1**; *their* 30 must be > 0
	$t=1.7$ cao; nfww	**A1**	
10(a)(ii)	$\frac{31}{3}-\frac{\mathrm{e}^{their\,1.7}}{3}-10\mathrm{e}^{-their\,1.7}+$ $\left(\frac{31}{3}-\frac{\mathrm{e}^{their\,1.7}}{3}-10\mathrm{e}^{-their\,1.7}-\left(\frac{31}{3}-\frac{\mathrm{e}^2}{3}-10\mathrm{e}^{-2}\right)\right)$ soi	**M2**	**FT** *their* $t\neq 0$ **M1** for $\frac{31}{3}-\frac{\mathrm{e}^{their\,1.7}}{3}-10\mathrm{e}^{-their\,1.7}$ or 6.68
	6.846 to 6.85	**A1**	must have earned 3 or 4 marks in (a)(i)
10(b)(i)	Straight line from (0, 0) stopping at (5, 10) and reasonable parabola of correct curvature between (5, 10) and (8, 25)	**B2**	**B1** for one section correct

Question	Answer	Marks	Partial Marks
10(b)(ii)	[distance 0 to 5 = $0.5 \times 5 \times 10$ =] 25	**B1**	
	$\left[\frac{t^3}{3} - \frac{8t^2}{2} + 25t\right]_5^8$ or $[s =]\ \frac{t^3}{3} - \frac{8t^2}{2} + 25t + c$	**M2**	**M1** for any 2 terms correct
	$\frac{8^3}{3} - \frac{8\times8^2}{2} + 25\times8 - \left[\frac{5^3}{3} - \frac{8\times5^2}{2} + 25\times5\right]$ or $25 = \frac{5^3}{3} - \frac{8\times5^2}{2} + 25\times5 + c$ and $\frac{8^3}{3} - \frac{8\times8^2}{2} + 25\times8 - \frac{125}{3}$	**M1**	**dep** on at least **M1** earned
	73 nfww	**A1**	
11(a)	Correct vector relationship involving $\overrightarrow{OC}$, $\overrightarrow{OA}$ and $\overrightarrow{OB}$: $\overrightarrow{OC} = \overrightarrow{OA} + 4(\overrightarrow{OB} - \overrightarrow{OA})$ or $\overrightarrow{OC} = \overrightarrow{OB} + 3(\overrightarrow{OB} - \overrightarrow{OA})$ or $\frac{1}{4}\overrightarrow{OC} - \frac{1}{4}\overrightarrow{OA} = \overrightarrow{OB} - \overrightarrow{OA}$ or $\overrightarrow{OC} - \overrightarrow{OA} = 4(\overrightarrow{OB} - \overrightarrow{OA})$ oe, soi	**M1**	
	Correct unsimplified vector expression for $\overrightarrow{OC}$ correctly shown to be the **given answer** 4**b** – 3**a**	**A1**	Must be convinced

Question	Answer	Marks	Partial Marks
11(b)	$\overrightarrow{DC} = 4\mathbf{b} - 3\mathbf{a} - \frac{3}{5}\mathbf{a}$ oe, soi, isw or $\overrightarrow{CD} = 3\mathbf{a} - 4\mathbf{b} + \frac{3}{5}\mathbf{a}$ oe, soi, isw	**B1**	
	$\overrightarrow{OE} = \frac{3}{5}\mathbf{a} + \lambda(4\mathbf{b} - 3\mathbf{a} - \frac{3}{5}\mathbf{a})$ oe or $\overrightarrow{OE} = 4\mathbf{b} - 3\mathbf{a} + \mu(3\mathbf{a} - 4\mathbf{b} + \frac{3}{5}\mathbf{a})$ oe	**B1**	**FT** *their* $\overrightarrow{DC}$ or $\overrightarrow{CD}$
	$\frac{3}{5} - 3\lambda - \frac{3}{5}\lambda = 0$ or better or $-3 + 3\mu + \frac{3}{5}\mu = 0$	**M1**	
	$\lambda = \frac{1}{6}$ oe or $\mu = \frac{5}{6}$ oe	**A1**	
	$\overrightarrow{\mathrm{OE}} = \frac{4}{6}\mathbf{b}$ oe	**A1**	**FT** $4 \times$ *their* λ or $4 - 4 \times$ *their* μ
12(a)	$-5 + 4d = 7$ soi	**M1**	
	$d = 3$	**A1**	
	$\frac{40}{2}\{2(-5) + (40-1) \times \textit{their } d\}$ oe	**M1**	
	2140	**A1**	
12(b)	Correct method for finding a or r e.g. $\frac{ar^5}{ar^2} = \frac{0.064}{8}$ oe	**M1**	
	$r = 0.2$ oe, nfww	**A1**	
	$a = 200$ nfww	**A1**	
	$\left[S_\infty = \frac{200}{1-0.2} =\right] 250$	**B1**	**FT** *their* a and r, providing $\lvert \textit{their } r \rvert < 1$